

Assessment of regularization techniques used in electrocardiographic imaging

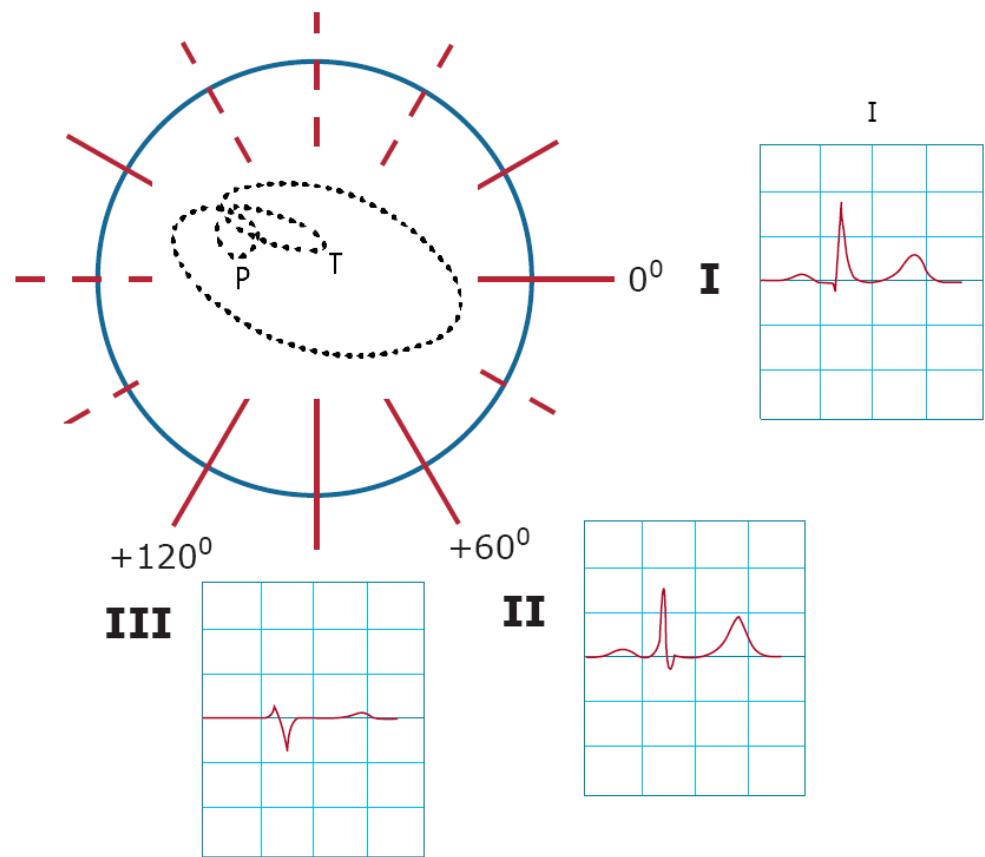
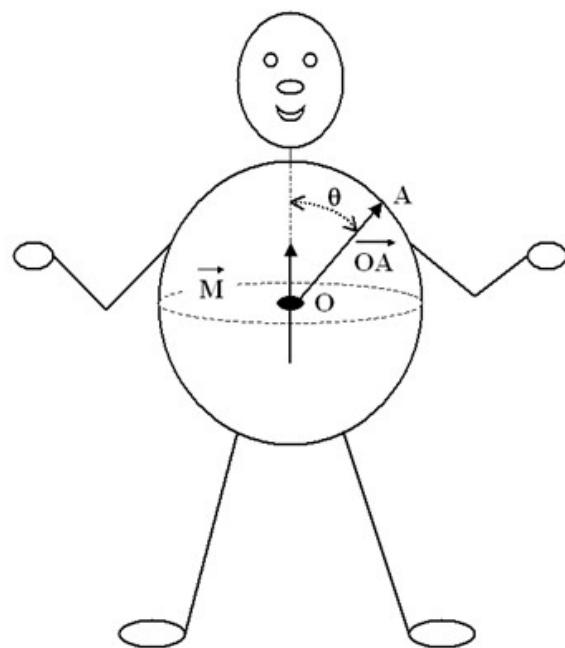
Ocenitev regularizacijskih tehnik pri
elektrokardiografskem slikanju

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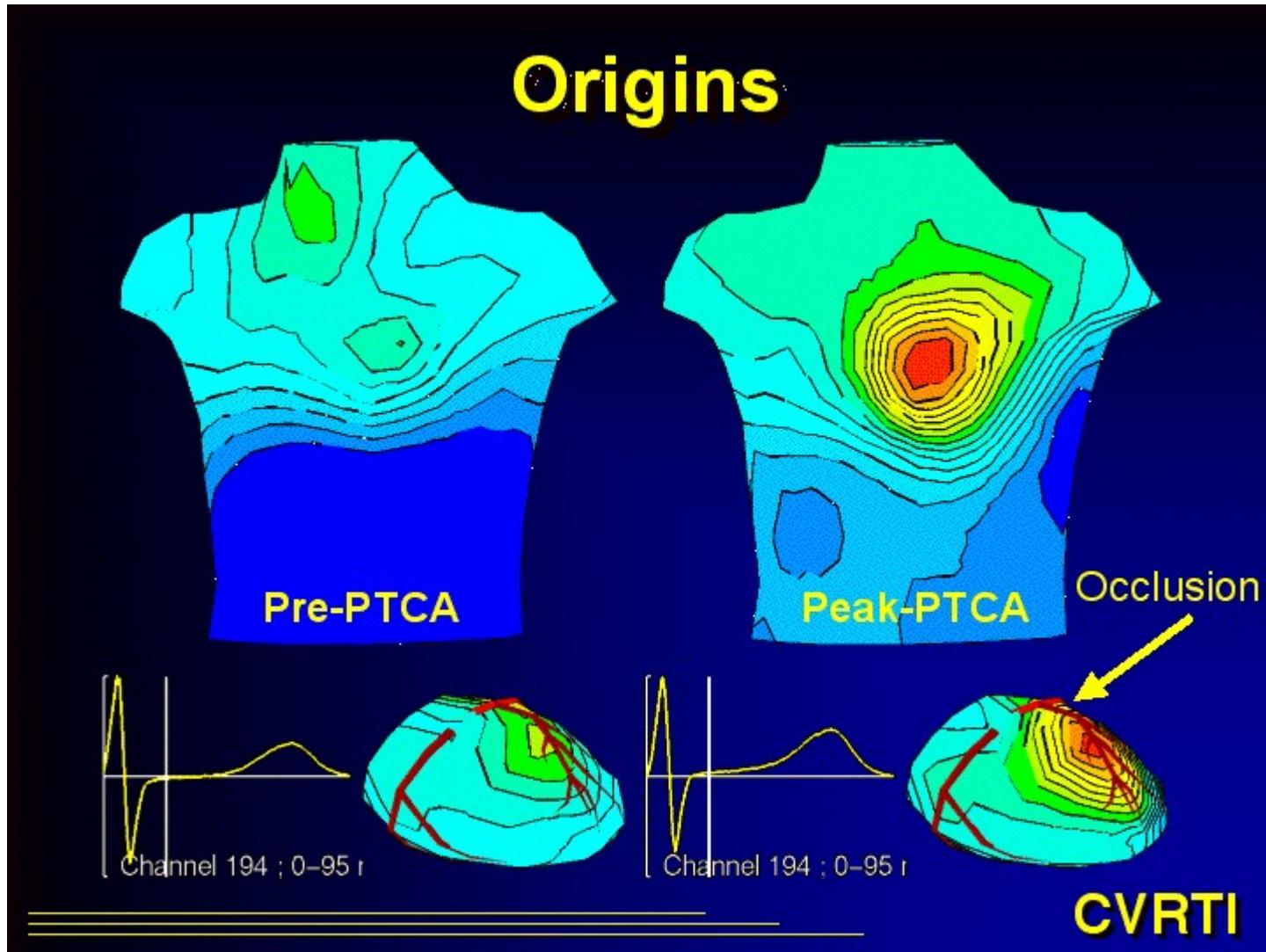
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Klinična elektrokardiografija = “srce kot dipol”

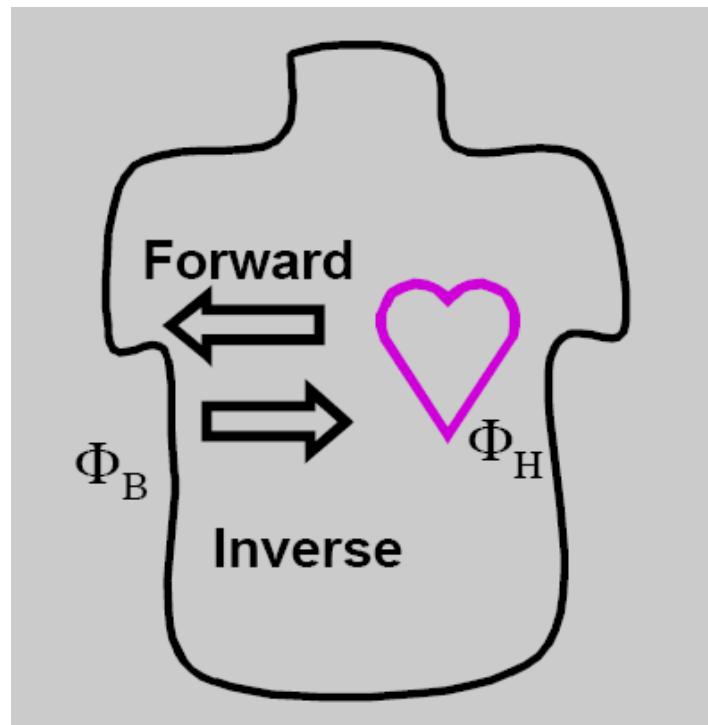


Raziskovalna elektrokardiografija = “srce kot porazdelitev potencialov na epikardu”



ECGI (Electrocardiographic imaging) - elektrokardigrafsko slikanje: $\Phi_B = A \Phi_H$

- Tehnično ozadje:
 - Problem formuliramo v obliki diferencialnih enačb
 - Numerične rešitve
 - Boundary element method – BEM
 - Finite element method – FEM
 - Regularizacija inverzne rešitve
 - ECGI je slabo pogojen problem



Regularizacijske tehnike

A. Tihonov-e regularizacijske metode

$$\min_{\Phi_H} \{ \| \Phi_B - A\Phi_H \|_2 + \lambda^2 \| \Lambda\Phi_H \|_2 \}, \quad \lambda - \text{regularizacijski parameter},$$

Λ – regularizacijski operator (Z=Identiteta, F=Gradient, S=Laplace)

B. Iterativne metode

C. Ne-kvadratne metode

$$\min_{\Phi_H} \{ \| \Phi_B - A\Phi_H \|_2 + \lambda^2 \| \Lambda\Phi_H \|_1 \}$$

	Abbr.	Short description	Ref.
A	ZOT	Zero-order Tikhonov	[6,7]
B	ZCG	Zero-order Conjugate Gradient	[8]
B	ZLSQR	Zero-order LSQR	[9]
B	TSVD	Truncated Singular Value Decomposition	[10]
B	Nu	v-method	[10]
A	FOT	First-order Tikhonov	[11]
B	FCG	First-order Conjugate Gradient	[8]
B	FLSQR	First-order LSQR	[9]
C	TV	Total Variation	[11]
C	LASSO	Least Absolute Selection and Shrinkage Operator	[12]

- [6] A. Tikhonov, V. Arsenin. Solution of Ill-Posed Problems. Washington, DC: Winston, 1977.
- [7] D.H. Brooks, F.A. Ghandi, R.S. MacLeod. Inverse Electrocardiography by Simultaneous Imposition of Multiple Constraints, IEEE Trans. Biomed. Eng. **46**: 3-18, 1999.
- [8] M. Hanke. Conjugate Gradient Type Methods for Ill-Posed Problems. Harlow: Longman Scientific & Technical, 1995.
- [9] C.C. Paige, M.A. Saunders. LSQR: An algorithm for sparse linear equations and sparse least squares, ACM Transactions on Mathematical Software **8**: 43-71, 1982.
- [10] P.C. Hansen. Rank-Deficient and Discrete Ill-Posed Problems. Philadelphia: SIAM, 1998.
- [11] G. Subham, Y. Rudy. Application of L1-Norm Regularization to Epicardial Potential Solution of the Inverse Electrocardiography Problem, Annals Biomed. Eng. **37**: 902-912, 2009.
- [12] M. Schmidt. Least Squares Optimization with L1-Norm Regularization, Project Report, University of British Columbia, 2005.

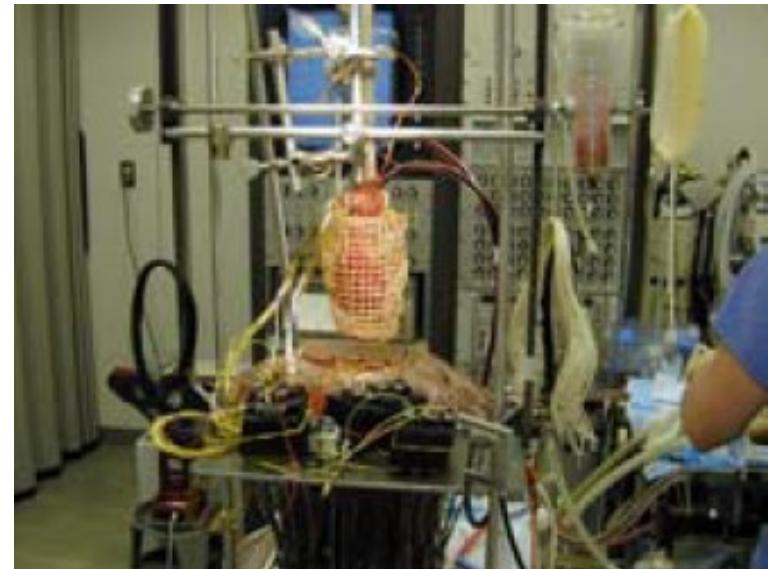
Motivacija: Primerjava različnih regularizacijskih tehnik na istem modelu prostorskega prevodnika z enakim izvorom v srcu

Prvi korak (CVRTI – Utah, ZDA)

- Meritve električnih potencialov na cilindrični mreži 602 odvodov okoli
- Izoliranega pasjega srca (normalno utripanje srca)

Drugi korak:

- Izračun potencialov na trikotizirani površini telesa v 771 točkah

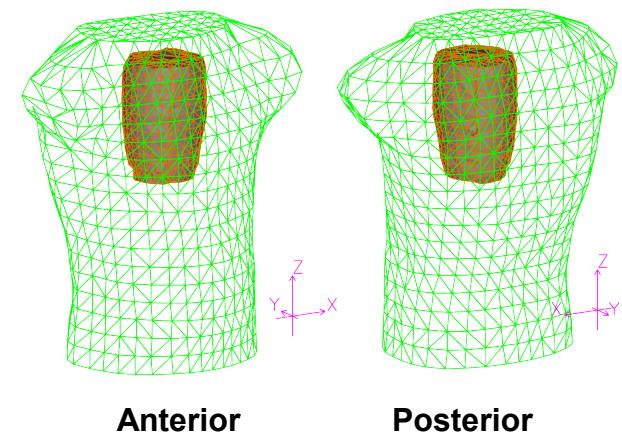


Tretji korak:

- Inverzni izračun potencialov na kletki iz potencialov na površini telesa (uporaba različnih regularizacij)

Četrти korak:

- Ovrednotenje rezultatov



Ovrednotenje

- Natančnost rešitve smo izrazili z relativno napako in koreacijskim koeficientom

$$RE = \frac{\|VE - VE_0\|_2}{\|VE_0\|_2}$$

$$CC = \frac{\text{cov}(VE, VE_0)}{\sigma_{VE} \sigma_{VE_0}}$$

kjer je VE_0 izmerjena in VE inverzno izračunana porazdelitev električnih potencialov na površini kletke (epikarda).

Ključna vprašanja

K? #1: Skupina A (Tihonove) vs. B (iterativne) vs. C (nekvadratne) regularizacijske tehnike

K? #2: Z (ničti) vs. F (prvi) vs. S (drugi) red regularizacije.

K? #3: BEM vs. FEM izračun potencialov na površini telesa.

Rezultati – začetna faza QRS kompleksa

Table 2: Relative errors (RE) za rekonstrukcije map $5 (Q_5)$, $10 (Q_{10})$, $15 (Q_{15})$ ms po začetku Q; Q_{pk} predstavlja vrh Q-vala.

	ZOT	FOT	SOT	ZCG	FCG	SCG	ZLSQR	FLSQR	SLSQR	TSVD	v	FTV	STV	LASSO
Q_5	0.32	0.22	0.22	0.32	0.25	0.25	0.32	0.25	0.25	0.33	0.32	0.23	0.22	0.36
Q_{10}	0.26	0.11	0.10	0.26	0.11	0.11	0.26	0.11	0.11	0.27	0.26	0.15	0.12	0.26
Q_{15}	0.30	0.18	0.16	0.26	0.19	0.15	0.26	0.19	0.15	0.27	0.27	0.14	0.13	0.27
Q_{pk}	0.49	0.43	0.39	0.40	0.45	0.38	0.40	0.45	0.38	0.44	0.45	0.31	0.25	0.40
	A - Tikhonov			B – Iterativne metode								C - nekvadratne		

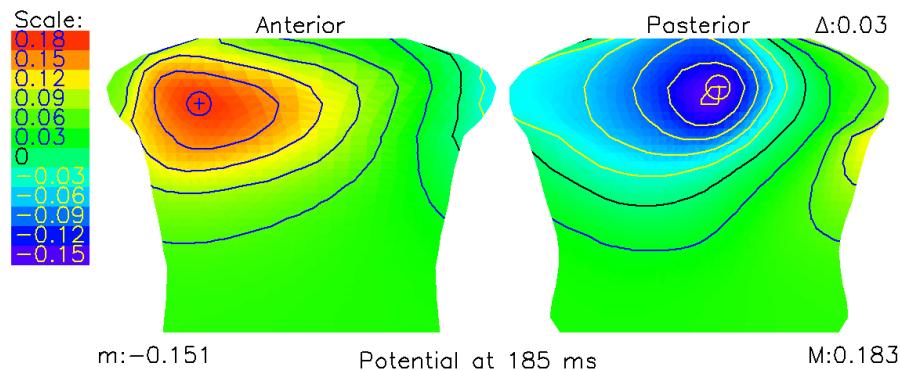
Rezultati – standardne referenčne točke (P,R,S,T)

Table 3. Relativne napake (RE) za rekonstrukcije potencialnih map v standardnih točkah srčnega utripa (vrhovi P, R, S, and T valov).

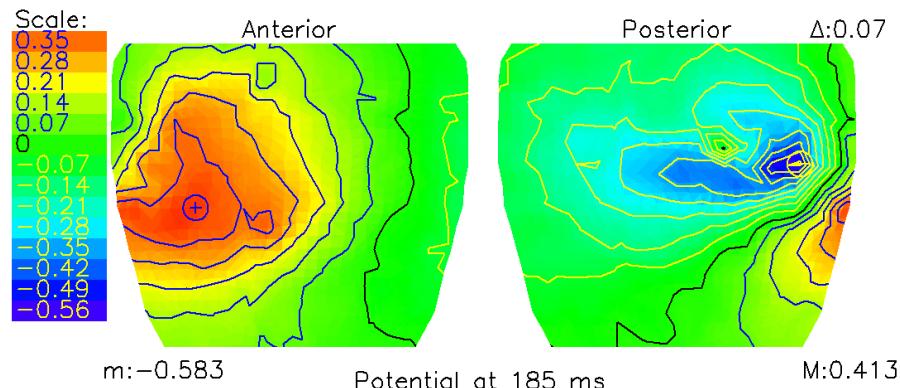
	ZOT	FOT	SOT	ZCG	FCG	SCG	ZLSQR	FLSQR	SLSQR	TSVD	v	FTV	STV	LASSO
P	0.47	0.43	0.42	0.47	0.45	0.45	0.47	0.45	0.45	0.51	0.48	0.37	0.41	0.45
R	0.45	0.40	0.39	0.40	0.40	0.38	0.40	0.40	0.38	0.42	0.43	0.35	0.33	0.40
S	0.48	0.42	0.40	0.47	0.45	0.44	0.47	0.45	0.44	0.50	0.49	0.37	0.40	0.45
T	0.27	0.16	0.16	0.26	0.16	0.16	0.26	0.16	0.16	0.27	0.26	0.17	0.16	0.26
	A - Tikhonov			B – Iterativne metode								C - nekvadratne		

Primer: Potencialne mape 5 ms po začetku Q-vala (Q_5)

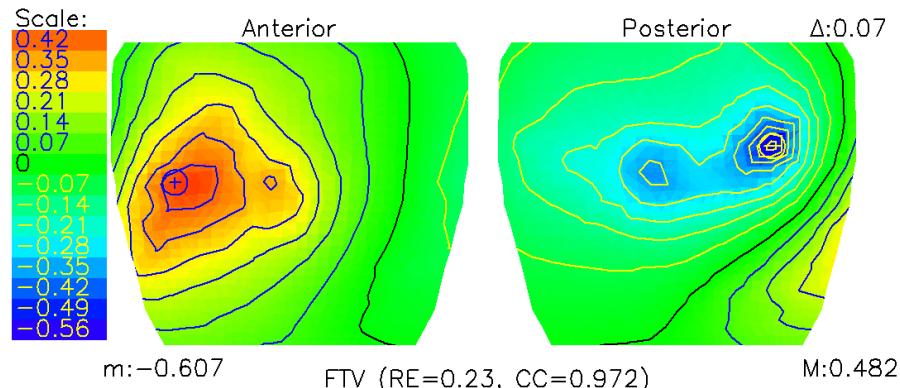
A Izračunana mapa (BEM)



B Izmerjena mapa na kletki

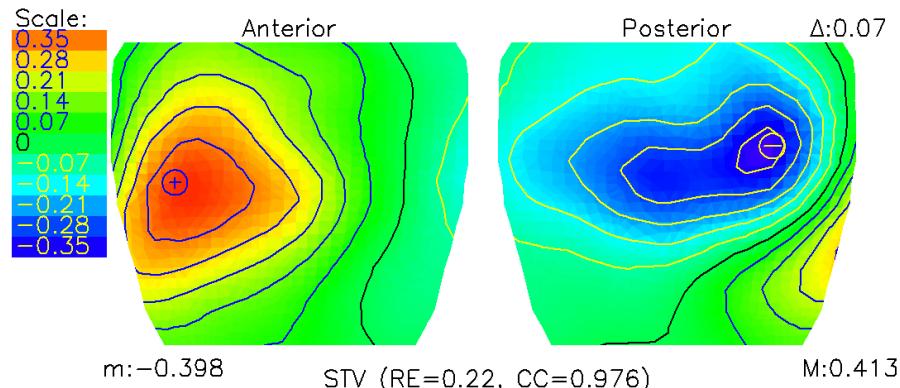


C Inverzno izračunana mapa (FTV)



Prvi red: gradientni operator

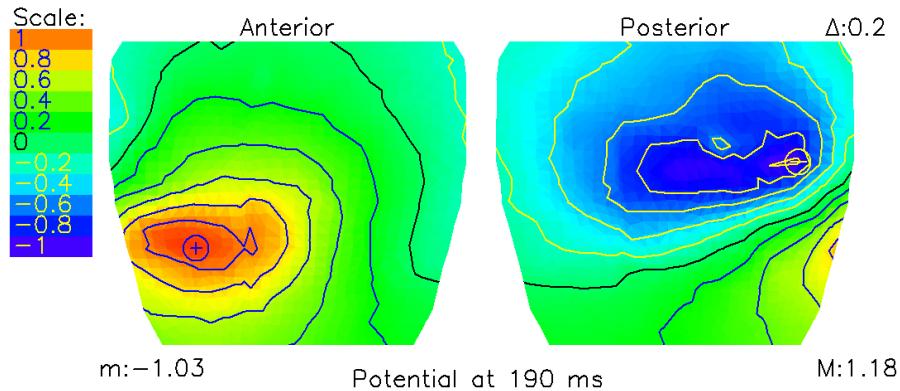
D Inverzno izračunana mapa (STV)



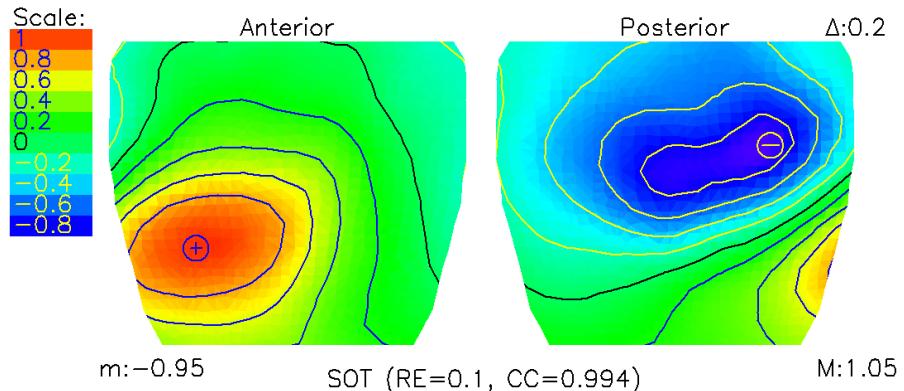
Drugi red: Laplaceov operator

Q_{10} – začetek QRS kompleksa

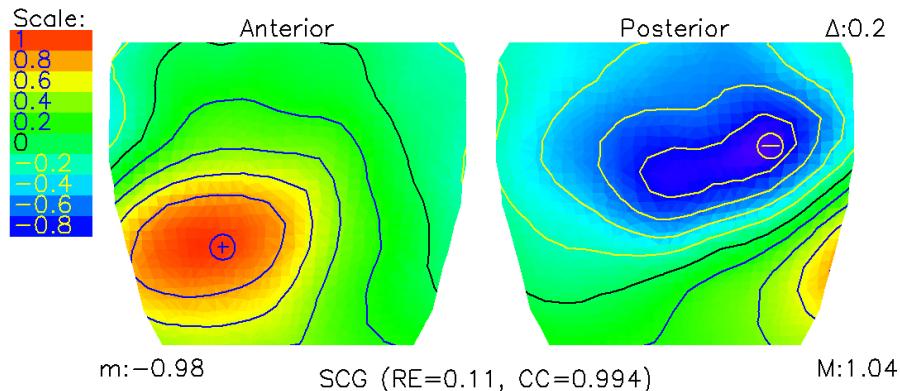
Measured



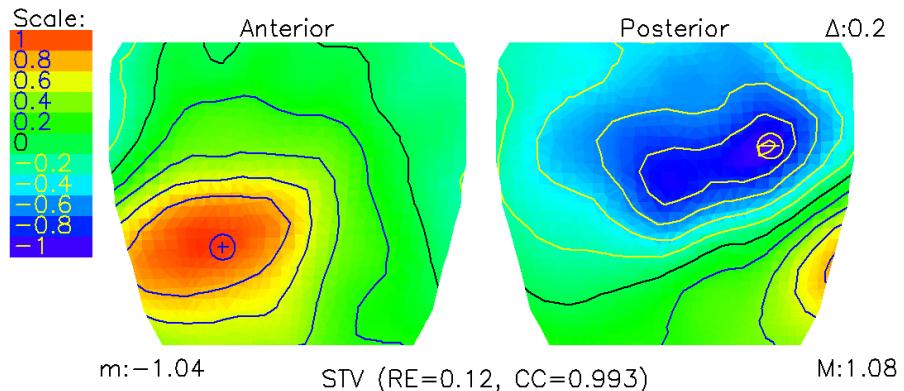
Group A



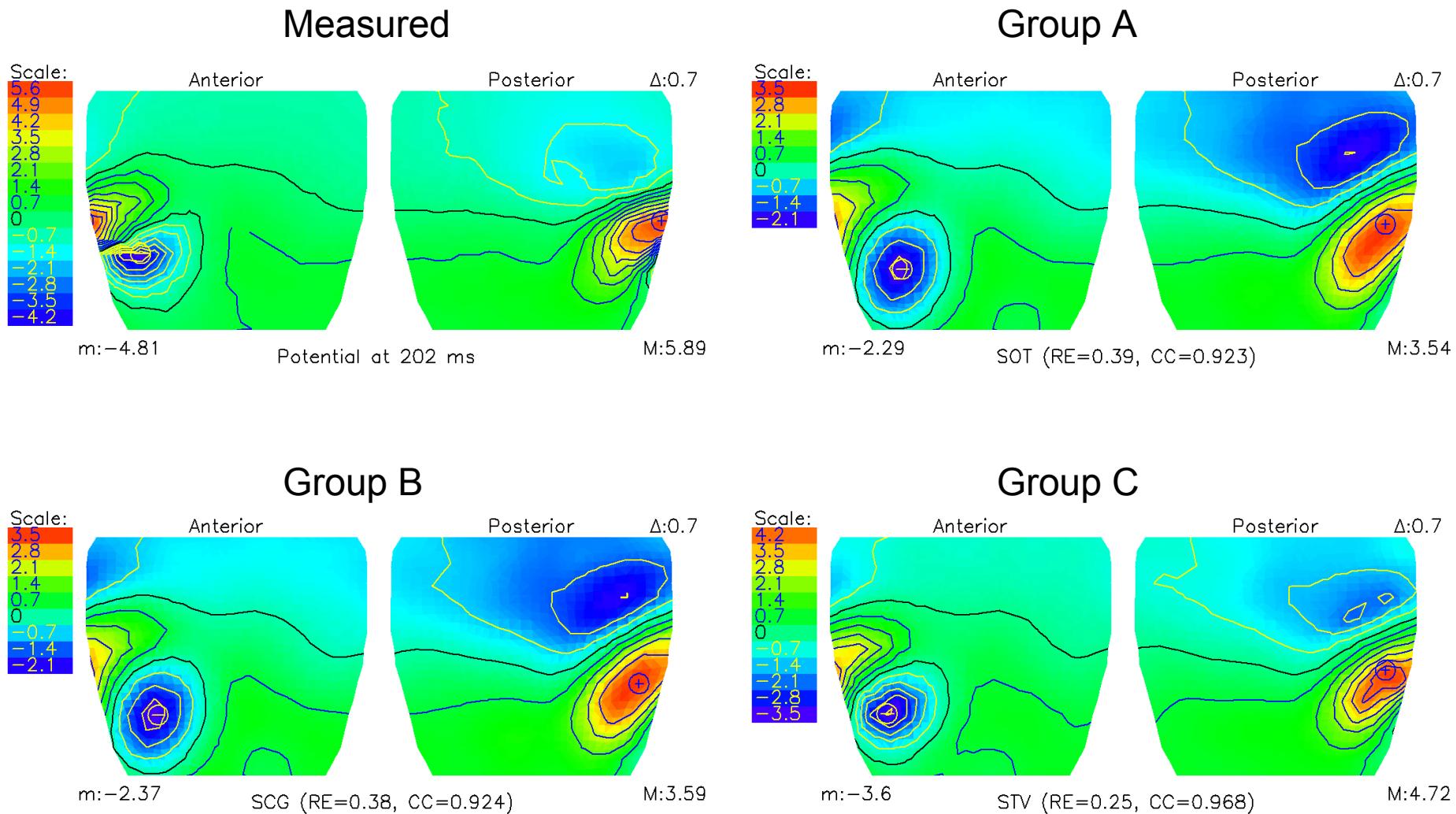
Group B



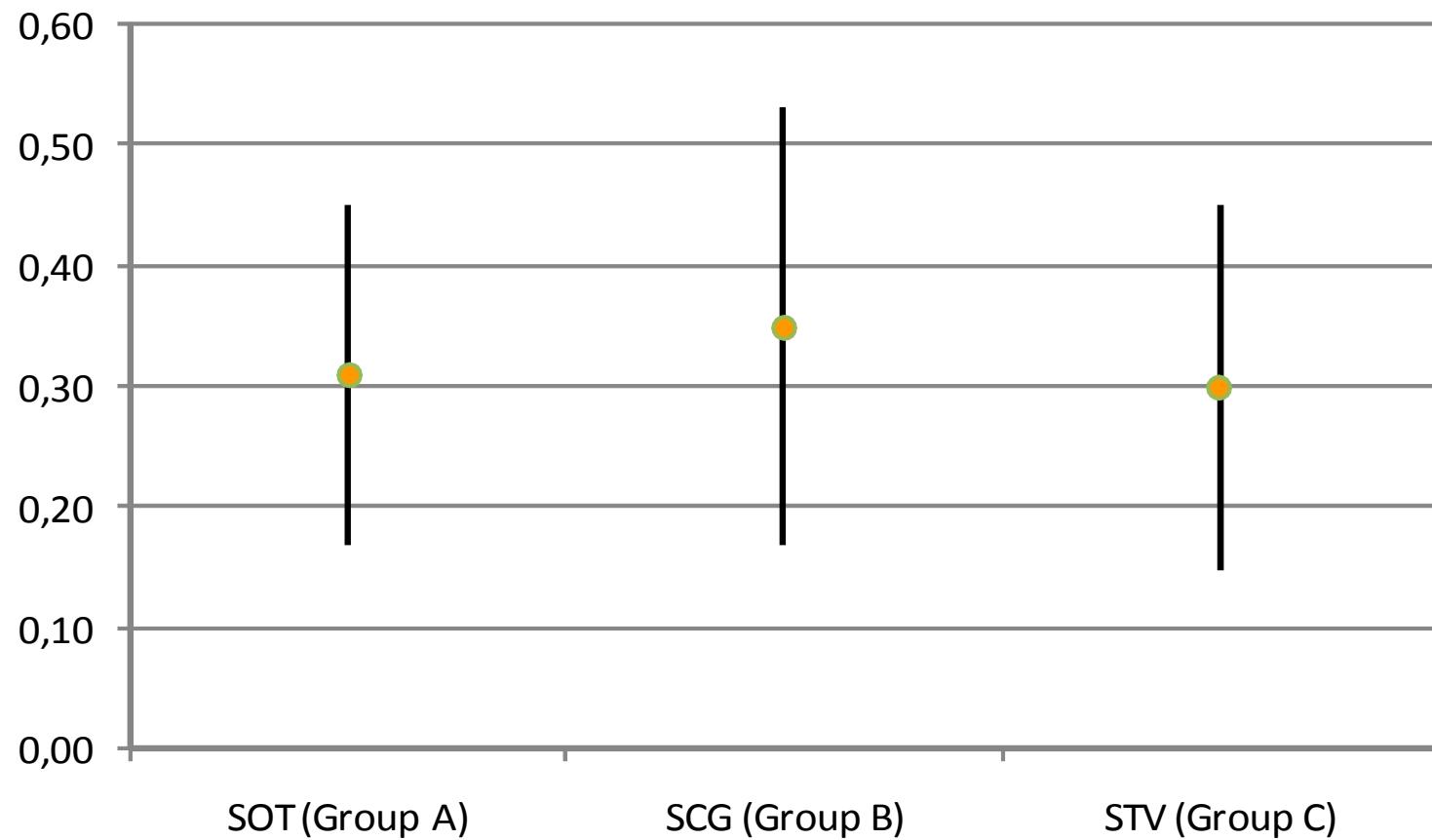
Group C

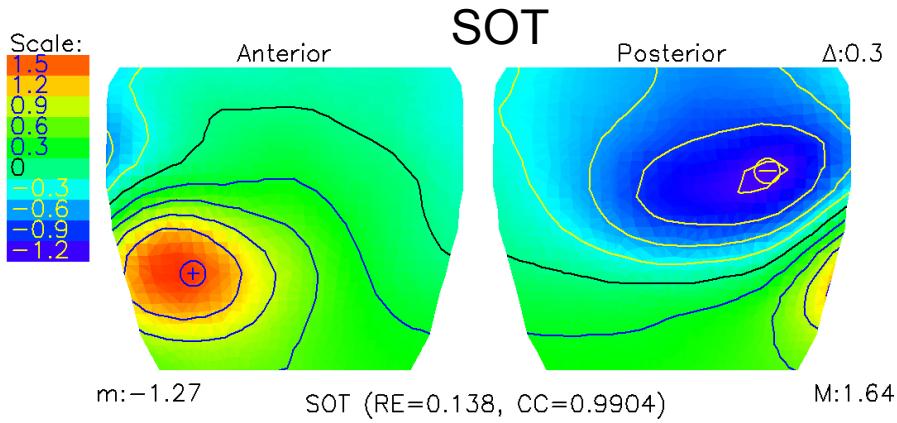
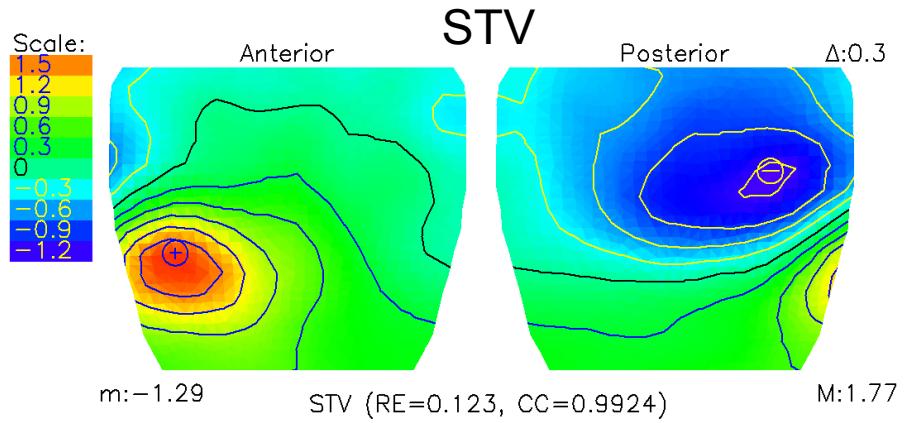
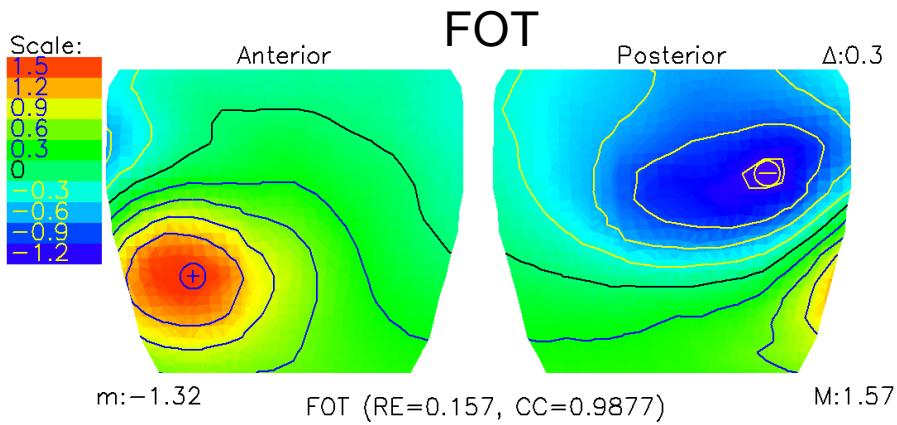
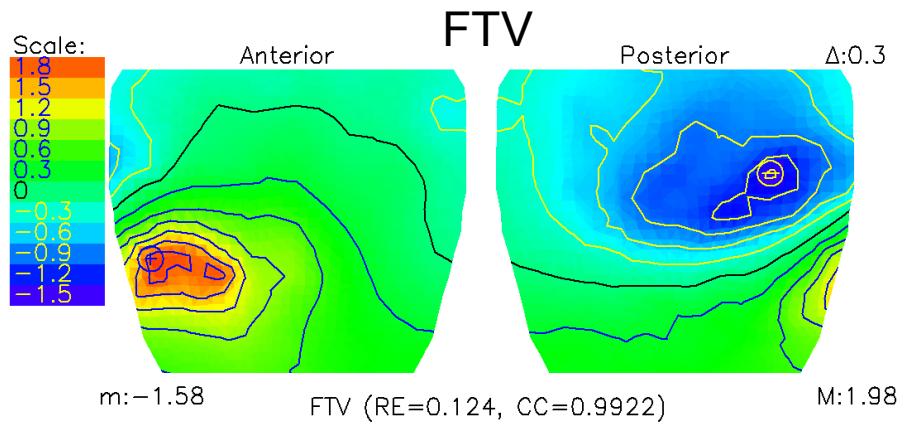
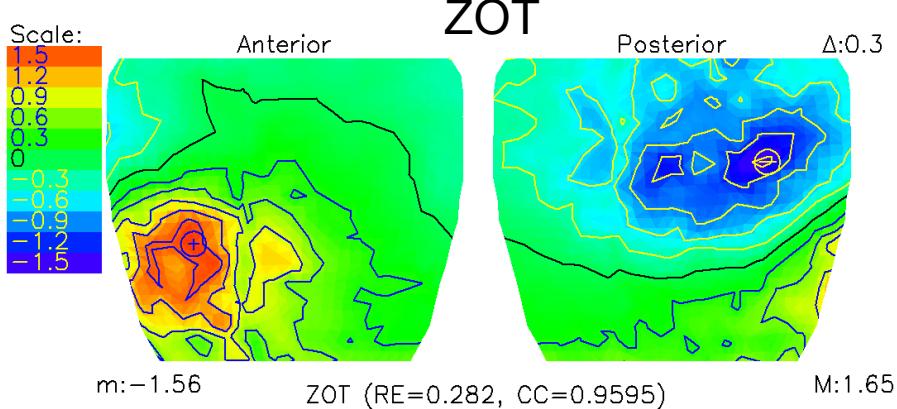
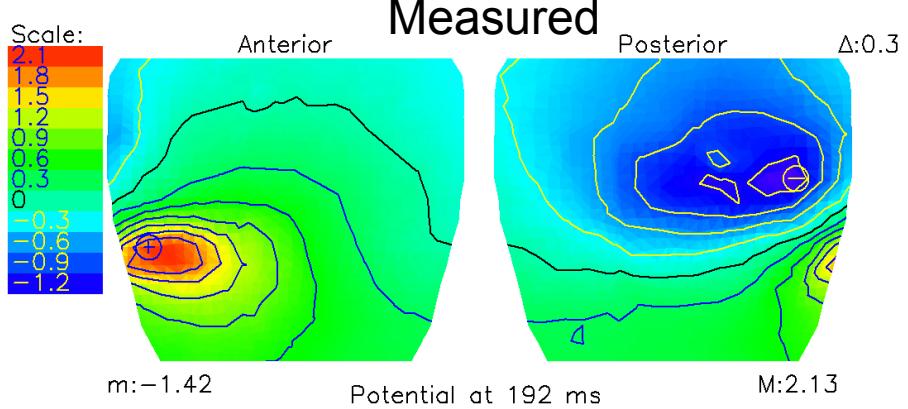


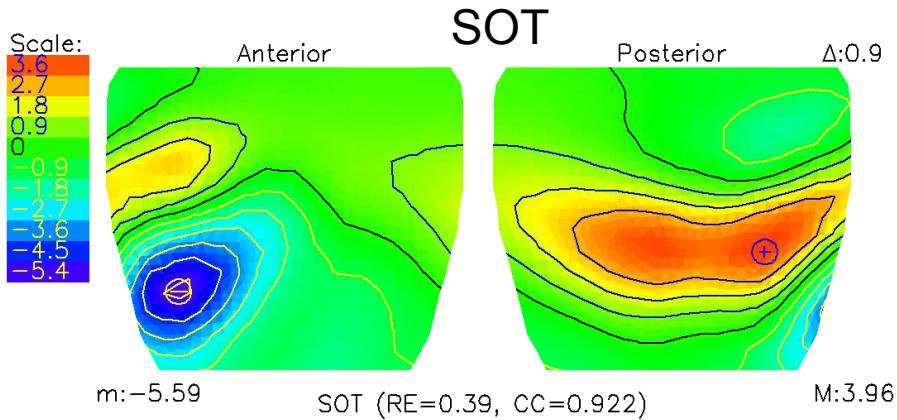
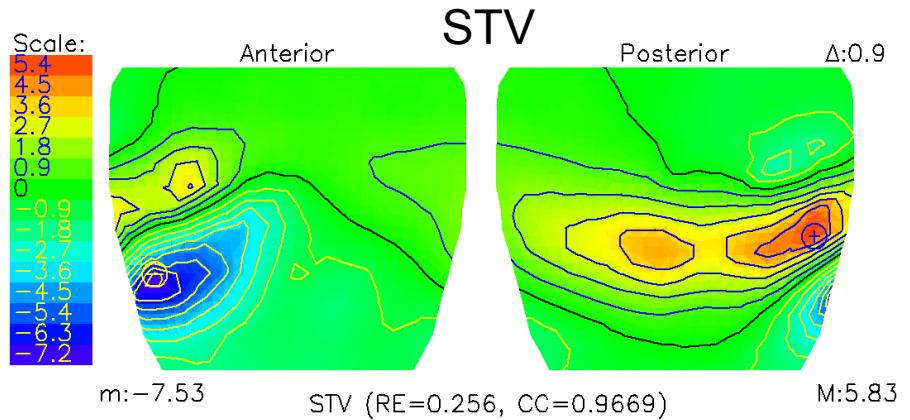
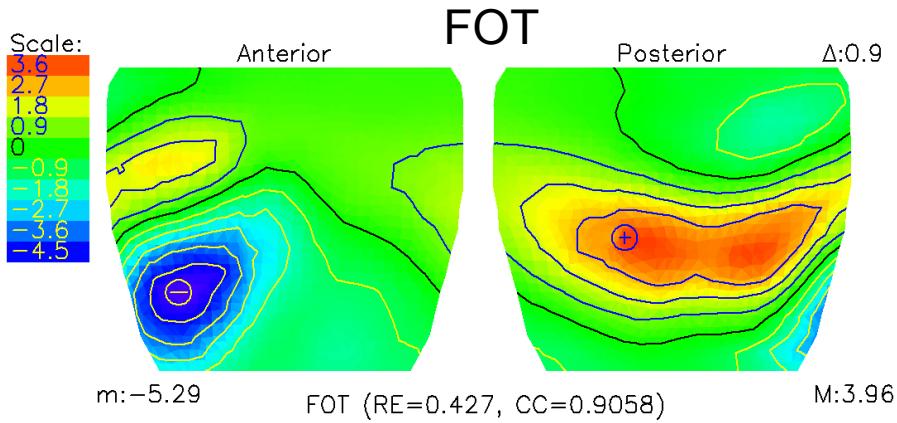
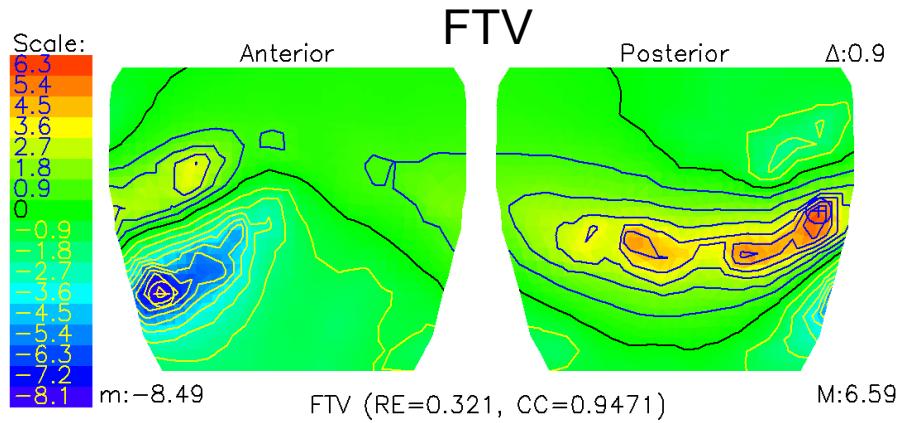
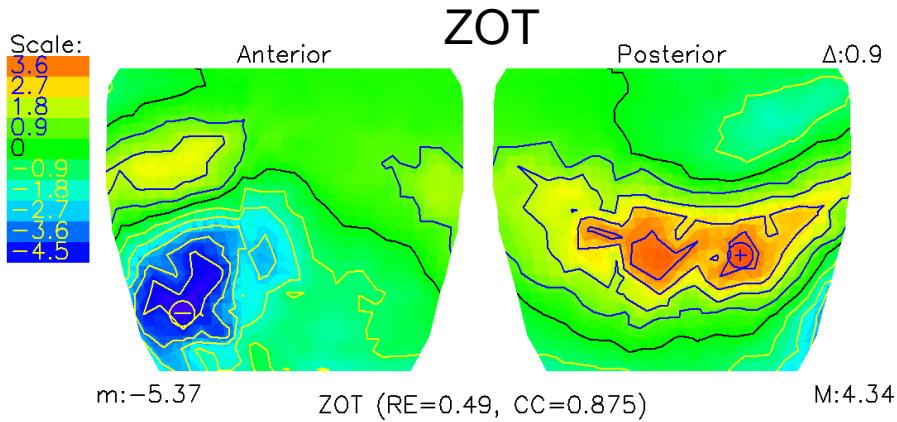
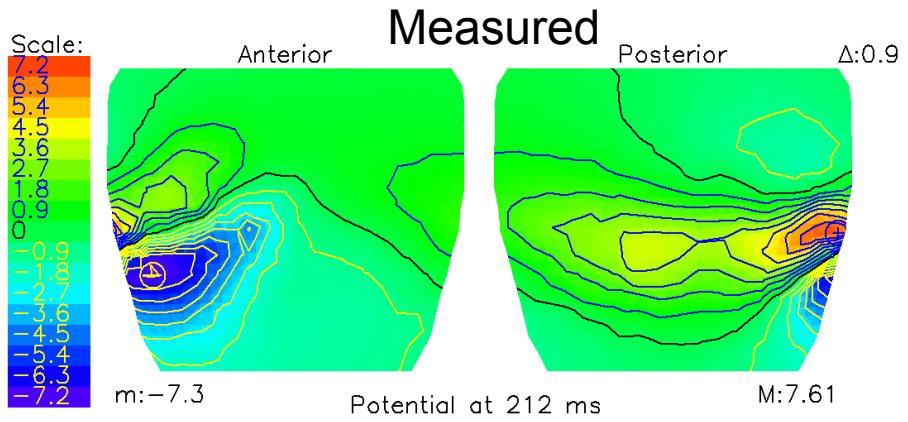
$Q_{pk} - vrh$ Q-vala



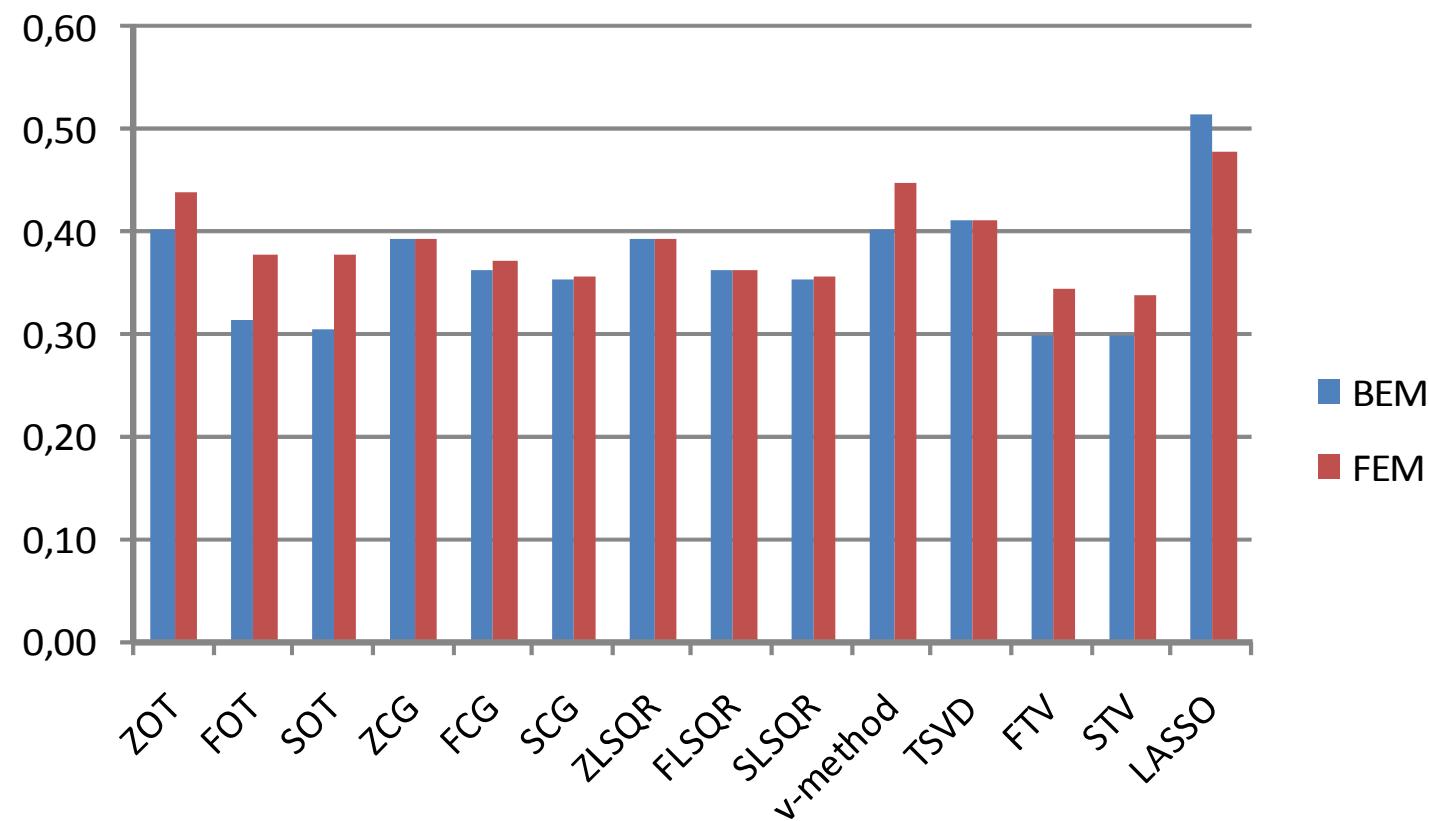
Povprečna relativna napaka ($RE \pm SD$) za celotni srčni utrip ($n = 484$)







Povprečne relativne napake (RE) za celotni srčni utrip (n = 484)



Glavni zaključki

- Metode totalne variacije (FTV, STV) zgledajo najbolj robustne
- Operatorji drugega reda bolje zajamejo kompleksne prostorske vzorce
- Za izotropni homogeni prostorski prevodnik je BEM boljša od FEM

Hvala za pozornost

