ECGI - Analytically solvable Sphere-Cage volume conductor model

In order to test simulated data calculated with the BEM and ECGI transfer matrix, we created a sphere, which replaced the tank surface. To find an optimal sphere, we wrote a program *get_opt_sphere*, which found the best fitting sphere using all or selected nodes on the tank surface. Fig. 1 show results for different node selection on the tank surface, which were determined by vertical range of z-coordinate. For each selection, a position and a radius of a best fitting sphere were calculated. Nodes from the tank surface were projected on the sphere and a relative fit error was determined as a relative difference between coordinates of N_{sel} selected nodes ($\vec{r}_i = (x_i, y_i, z_i)$) on the tank and coordinates of projected nodes on the sphere ($\vec{R'}_i = (x'_i, y'_i, z'_i)$):

$$\text{Fit error} = \sqrt{\frac{\sum_{i=1}^{N_{\mathrm{sel}}}((x'_i - x_i)^2 + (y'_i - y_i)^2 + (z'_i - z_i)^2)}{\sum_{i=1}^{N_{\mathrm{sel}}}(x_i^2 + y_i^2 + z_i^2)}} \tag{1}$$

Table 1 displays some evaluation paramters logged during *get_opt_sphere* procedure: c – center of sphere (Cartesian coordinates in mm), r – sphere radius in mm, relative fit error (1), mean dist – mean distance ± standard deviation and max error - maximal distance between nodes on the tank surface and corresponding nodes on the sphere. The smallest fit error 0.059 and mean distance 16.1±12.2 mm were obtained when only upper part of nodes ($z \in [200, 401]$) were selected. The smallest max error 56.4 mm was obtained for nodes selected on range $z \in [70, 401]$. Combining all evaluation parameters and from visual inspection of results in Fig. 1, we chose a sphere obtained by selection of nodes on range $z \in [150, 401]$ as a model for comparing BEM and analytical calculation. For this sphere, we got fit error 0.064, max error 57.7 mm and mean distance 16.8±12.2 mm.

Table 1: Log data (from *get_opt_sphere*)

Input new z-coordinate range, current choice is (0,400.9) : 0,401 Fit_result: c=(-19.94,41.53,203.42), r=168.44 Fit error=0.15389, mean dist=36.668 ± 18.038 max error=69.508 at node 116 (31.7,-42.8,200.5) show_obj: save to tank_sphere_fit.tri	Input new z-coordinate range, current choice is (0,400.9) : 50,401 Fit_result: c=(-21.288,43.73,243.57), r=151.06 Fit error=0.097638, mean dist=24.625 ± 15.183 max error=59.42 at node 395 (66.4,-29.5,66.8) show_obj: save to : tank_sphere_z50.tri
Input new z-coordinate range, current choice is (0,400.9) : 70,401 Fit_result: c=(-21.405,44.257,252.08), r=147.5 Fit error=0.086201, mean dist=21.876 ± 14.281 max error=56.41 at node 645 (153.5,34,356.4) show_obj: save to tank_sphere_z70.tri	Input new z-coordinate range, current choice is (0,400.9) : 90,401 Fit_result: c=(-21.486,44.886,260.04), r=144.2 Fit error=0.076841, mean dist=19.725 ± 13.378 max error=56.531 at node 621 (165,50.4,334.1) show_obj: save to : tank_sphere_z90.tri
Input new z-coordinate range, current choice is (0,400.9) : 120,401 Fit_result: c=(-21.51,45.509,267.82), r=140.95 Fit error=0.069146, mean dist=17.925 ± 12.729 max error=57.053 at node 621 (165,50.4,334.1) show_obj: save to tank_sphere_z120.tri	Input new z-coordinate range, current choice is (0,400.9) : 150,401 Fit_result: c=(-21.499,46.078,275.06), r=137.93 Fit error=0.06368, mean dist=16.813 ± 12.173 max error=57.736 at node 621 (165,50.4,334.1) show_obj: save to : tank_sphere_z150.tri
Input new z-coordinate range, current choice is (0,400.9) : 200,401 Fit_result: c=(-21.671,46.973,286.86), r=132.87 Fit error=0.05887, mean dist=16.062 ± 12.176 max error=59.721 at node 621 (165,50.4,334.1) show_obj: save to tank_sphere_z200.tri	Input new z-coordinate range, current choice is (0,400.9) : 210,401 Fit_result: c=(-21.554,47.003,291.19), r=131.16 Fit error=0.058997, mean dist=16.561 ± 12.399 max error=60.299 at node 621 (165,50.4,334.1) show_obj: save to : tank_sphere_z210.tri

Sphere-Cage model has analytical solution and can be used for evaluation of BEM and ECGI potential calculations. We used sphere triangulation based on a 3-times refined icosahedron with 1280 triangles and 642 nodes. Cage surface was the same as in the Tank-Cage model. Icosahedron consists of 20 equilateral triangles and 12 nodes. In each refinement step, each triangle is divided into 4 equilateral triangles by putting additional nodes in the middle of each triangle side. These additional nodes are expanded to the sphere surface.

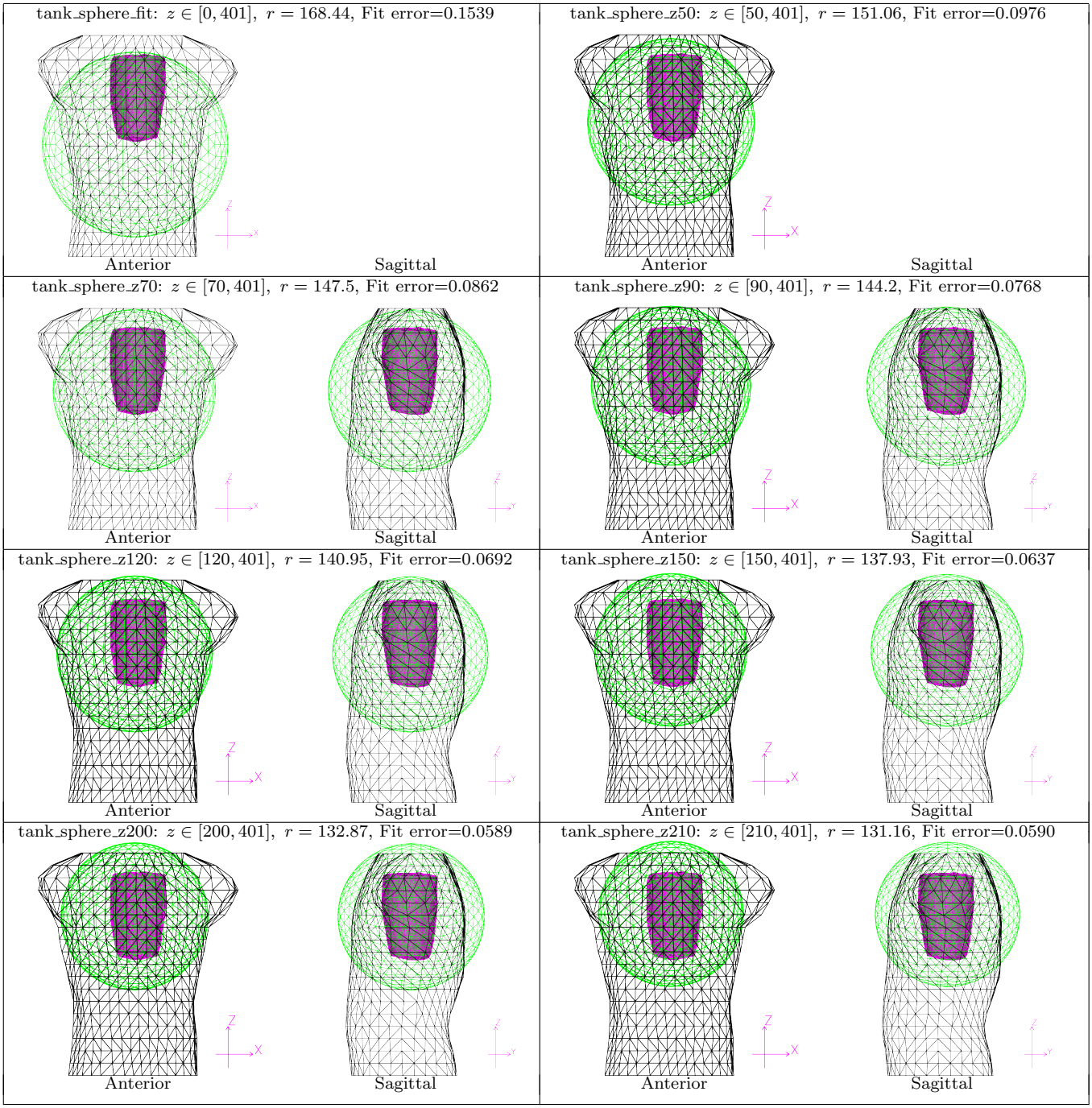

Fig. 1: Anterior and sagittal views of optimal spheres fitted to different selections of nodes on the tank surface. For each selection, z-interval in mm of nodes taken into account, obtained sphere radius in mm and relative fit error are displayed, respectively. The smallest fit error was obtained when only upper part of nodes ($z \in [200, 401]$) were selected.